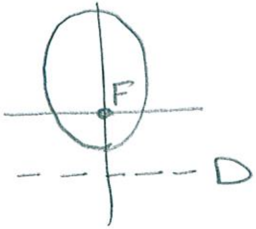
An ellipse has a focus at the pole and vertices with **rectangular** co-ordinates $(0, -4)$ and $(0, 7)$. **SCORE: _____ / 20 PTS**

[a] Give polar co-ordinates for the vertices, using positive values of r and θ.

$\left(4, \frac{3\pi}{2}\right)$ 2 , $\left(7, \frac{\pi}{2}\right)$ 2



[b] Find the **polar** equation of the ellipse.

$$r = \frac{ep}{1 - e\sin\theta}$$

$$4 = \frac{ep}{1+e} \quad 2 \qquad 7 = \frac{ep}{1-e} \quad 2$$

$$ep = 4 + 4e = 7 - 7e \quad 1$$

$$11e = 3$$

$$e = \frac{3}{11} \quad 1$$

$$\frac{3}{11}p = 4 + \frac{12}{11} \quad 1$$

$$3p = 44 + 12$$

$$p = \frac{56}{3} \quad 1$$

$$r = \frac{\boxed{\frac{56}{11}}}{1 - \boxed{\frac{3}{11}}\sin\theta} \quad 1 \quad 1$$

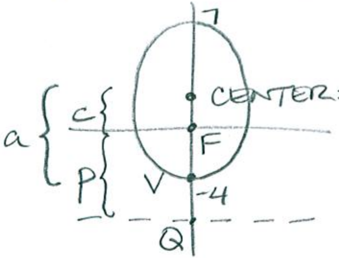
$$r = \frac{56}{11 - 3\sin\theta}$$

2 2

2 SIMPLIFIED

[b] Find the **polar** equation of the ellipse.

ALTERNATE SOLUTION



$\text{CENTER} = \left(0, \frac{7+-4}{2}\right) = \left(0, \frac{3}{2}\right)$

$c = \boxed{\frac{3}{2}}$ 2

$a = \frac{3}{2} - -4 = \boxed{\frac{11}{2}}$ 2

$e = \frac{c}{a} = \boxed{\frac{3}{11}}$ 2

$e = \frac{VF}{VQ}$

$\frac{3}{11} = \frac{4}{p-4}$ 1

$3p - 12 = 44$

$p = \frac{56}{3}$ 1

$r = \frac{ep}{1 - e\sin\theta}$

$r = \frac{\boxed{\frac{56}{11}}\ 1}{1 - \boxed{\frac{3}{11}}\sin\theta}$ 1

$r = \frac{56}{11 - 3\sin\theta}$

2 2

2 SIMPLIFIED FINAL ANSWER

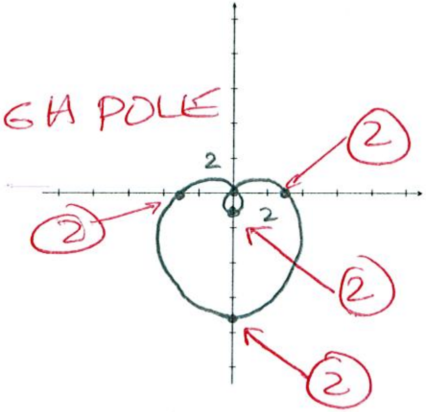
Consider the polar equation $r = 3 - 4\sin\theta$. **SCORE: _____ / 30 PTS**

[a] Sketch the graph of the polar equation using the shortcut process shown in lecture / PowerPoint.
Label the scale on your axes clearly.

θ	r	(x, y)
0	3	$(3,0)$
$\pi/2$	-1	$(0,-1)$
π	3	$(-3,0)$
$3\pi/2$	7	$(0,-7)$

5 SHAPE

2 GOES THROUGH POLE



[b] Convert the equation to rectangular.

3 $r^2 = 3r - 4r\sin\theta$ 3

$x^2+y^2 = 3\sqrt{x^2+y^2} - 4y$ 3 3

$x^2+y^2+4y = 3\sqrt{x^2+y^2}$

$(x^2+y^2+4y)^2 = 9x^2+9y^2$ 3

OR

$r = 3 - \frac{4y}{r}$ 3

$r^2 = 3r - 4y$ 3

3 $x^2+y^2 = 3\sqrt{x^2+y^2} - 4y$ 3

$x^2+y^2+4y = 3\sqrt{x^2+y^2}$

$(x^2+y^2+4y)^2 = 9x^2+9y^2$ 3

Consider the polar equation $r = \dfrac{40}{3 - 5\cos\theta}$.

SCORE: _____ / 25 PTS

[a] Find the equation of the directrix.

$$r = \dfrac{\frac{40}{3}}{1 - \frac{5}{3}\cos\theta} \quad 2$$

$$e = \frac{5}{3} \quad 2 \qquad ep = \frac{40}{3} \quad 1$$

$$\frac{5}{3}p = \frac{40}{3} \longrightarrow p = 8$$

$$x = -8 \quad 2 \; 2 \; 2$$

[b] Find the **rectangular** coordinates of the endpoints of **all** latera recta. **Do NOT convert the equation to rectangular.**

θ	r	(x,y)
0	-20	$(-20,0)$
$\pi/2$	$40/3$	$(0, 40/3)$
π	5	$(-5,0)$
$3\pi/2$	$40/3$	$(0, -40/3)$

4

CENTER $= \left(\frac{-20 + -5}{2}, 0\right) = \left(\frac{-25}{2}, 0\right)$ 2

FOCI $= \left(2 * \frac{-25}{2}, 0\right)$ AND $(0,0)$

$= (-25,0), (0,0)$

ENDS OF L.R.

$= \left(-25, \pm \frac{40}{3}\right), \left(0, \pm \frac{40}{3}\right)$

2 2 2 2

Consider the conic with rectangular equation $8x^2 - y^2 + 32x + 14y - 1 = 0$. **SCORE: _____ / 25 PTS**

[a] Find the co-ordinates of the focus/foci.

$$8x^2 + 32x - y^2 + 14y = 1$$

$$8(x^2 + 4x) - (y^2 - 14y) = 1$$

$$8(x^2 + 4x + 4) - (y^2 - 14y + 49) = \underbrace{1 + 32 - 49}_{3}$$

$$2 \quad \underbrace{8(x+2)^2}_{} \; \underbrace{-(y-7)^2}_{2} = \underbrace{-16}_{1}$$

$$3 \quad \underbrace{\frac{(y-7)^2}{16} - \frac{(x+2)^2}{2} = 1}_{}$$

$$c^2 = 16 + 2 = 18 \rightarrow c = \underbrace{3\sqrt{2}}_{3} \qquad (\underbrace{-2,}_{2} \; 7 \pm \underbrace{3\sqrt{2}}_{2})$$

[b] If the equation corresponds to a circle, find its radius.
If the equation corresponds to a parabola, find its directrix.
If the equation corresponds to an ellipse, find the endpoints of its minor axis.
If the equation corresponds to a hyperbola, find the equations of its asymptotes.

$$\overbrace{y - 7 = \underbrace{\pm}_{2} \underbrace{2\sqrt{2}}_{3}(x+2)}^{2}$$

Consider the polar equation $r = 1 + \sin 2\theta$.

SCORE: _____ / 20 PTS

The following symmetry tests do NOT indicate that the graph is symmetric:
$(-r, -\theta)$, $(-r, \pi-\theta)$ and $(r, \pi-\theta)$

$\theta = \pi/2$ POLAR AXIS $\theta = \pi/2$

[a] **Using the results above, along with the tests and shortcuts shown in lecture**, determine if the graph is symmetric over the polar axis, $\theta = \frac{\pi}{2}$ and/or the pole. Summarize your conclusions in the table on the right.
NOTE: Run as FEW tests as needed to prove your conclusions are correct.

POLAR AXIS: $(r, -\theta)$ 2 $r = 1 + \sin 2(-\theta)$

2 $r = 1 - \sin 2\theta$

POLE: $(-r, \theta)$ 2 $-r = 1 + \sin 2\theta$

2 $r = -1 - \sin 2\theta$

$(r, \pi+\theta)$ 2 $r = 1 + \sin 2(\pi+\theta)$

$r = 1 + \sin(2\pi + 2\theta)$

2 $r = 1 + \sin 2\theta$

Type of symmetry	Conclusion
Over the polar axis	NO CONCLUSION
Over $\theta = \frac{\pi}{2}$	NO CONCLUSION
Over the pole	SYMMETRIC

3

[b] Based on the results of part [a], what is the minimum interval of the graph you need to plot (before using reflections to draw the rest of the graph) ?

$\theta \in [0, \pi]$ OR $[-\frac{\pi}{2}, \frac{\pi}{2}]$ 5

Find the **rectangular** equation of the parabola with focus $(-7, 5)$ and directrix $x = 13$. **SCORE: _____ / 15 PTS**

VERTEX = $\left(\frac{-7+13}{2}, 5\right) = (3,5)$ 3

$p =$ DIRECTED DISTANCE FROM VERTEX TO FOCUS $= -7-3 = -10$

4 3 3

$(y-5)^2 = 4(-10)(x-3)$

$(y-5)^2 = -40(x-3)$

1 1

Name the shapes of the following graphs. **SCORE: _____ / 15 PTS**

[a] the graph with polar equation $r = \dfrac{13}{7 - \cos\theta}$

ELLIPSE $2\frac{1}{2}$

[b] the graph with polar equation $r = 13 + 7\sin\theta$ $2\frac{1}{2}$

LIMAÇON WITH DIMPLE

[c] the graph with equation $7 + 7x^2 - 13y + 13y^2 = 0$

ELLIPSE 2

[d] the graph with polar equation $\theta = 5$

LINE 2

[e] the locus of points in the plane that are half as far from $x = 7$ as they are from $(7, 7)$

HYPERBOLA 3

[f] the locus of points in the plane that are 7 units farther from $(7, -7)$ than they are from $(7, 7)$

HYPERBOLA 3